MALINENI LAKSHMAIAH WOMEN'S ENGINEERING COLLEGE
(AUTONOMOUS)
Department of Science & Humanities
I B.Tech I Semester Regular Examinations, FEBRUARY- 2024
LINEAR ALGEBRA & CALCULUS

MR23

Branches: ECE/CSE/IT/AI&ML/DS/AI&DS

Regd.No.

Time: 3 hours   Max. Marks: 70

Question Paper Consists of **Part-A and Part-B**
Answer **ALL** the Questions in **Part-A and Part-B**

		PART-A (10 X 2M = 20M)	CO	BTL	Marks
1	a	Find the value of k such that the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2	1	1	2M
	b	Are the systems of equations $x + y + z = 6, x + 2y + 2z = 14, x + 4y + 7z = 30$ consistent ?	1	1	2M
	c	If the Eigen values of A are -1,1,3 then find the Eigen values of Adj A.	2	1	2M
	d	Write the matrix form of the Quadratic form $x^2 + 2y^2 + 2z^2 - 4xy + 6zx + 10yz$.	2	1	2M
	e	State Lagrange's Mean Value Theorem.	3	2	2M
	f	What is the Maclaurin series expansion of the function $f(x) = \sin x$	3	1	2M
	g	Evaluate $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$ if $Z = \log(x^2 + y^2)$.	4	5	2M
	h	If $u = x^2 + y^2, x = at^2, y = 2at$ then find $\frac{du}{dt}$	4	1	2M
	i	Find $\iint x^2 \, dx \, dy$ over the region bounded by hyperbola $xy = 4, y = 0, x = 1, x = 4$.	5	1	2M
	j	Evaluate $\int_0^1 \int_1^2 \int_2^3 xyz \, dx \, dy \, dz$.	5	5	2M
		PART-B(5 X 10M = 50M)			
2	a	Verify Cauchy-Binet formula for $A = \begin{bmatrix} 1 & 3 & 5 & 6 \\ -3 & 5 & 7 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 7 & 8 \\ -9 & 2 \end{bmatrix}$	1	2	5M
	b	Find the values ' a' of ' b' such that the system of equations $x + y + z = 3, x + 2y + 2z = 6, x + ay + 3z = b$ have (i) No solution (ii) a unique solution (iii) infinite number of solutions.	1	1	5M
		Or			

3	a	Solve the system of equations $10x - 7y + 3z + 5w = 6, -6x + 8y - z - 4w = 5, 3x + y + 4z + 11w = 2, 5x - 9y - 2z + 4w = 7$ by using Gauss elimination method.	1	3	5M
	b	Solve the system of equations using Gauss-Seidel method $20x + y - 2z = 17, 3x + 20y - z = -18, 2x + 3y + 20z = 25$	1	3	5M
4	a	Find the Eigen values and Eigen vectors of the matrix A=$\begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$	2	1	5M
	b	Verify Cayley-Hamilton theorem for the matrix A=$\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ and find A^{-1}	2	2	5M
		Or			
5	a	Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy + 2zx - 2yz$ to the canonical form by orthogonal transformationsand find the rank, index, signature and nature of the quadratic form.	2	3	10M
6	a	Verify Rolle's mean value theorem for the function $f(x) = ((x-a)^m (x-b)^n$in$[a,b]$,where m and n arepositive integers.	3	2	5M
	b	Expand $f(x) = \log(1+x)$ in powers of by using Maclaurin's series and evaluate log 1.1	3	3	5M
		Or			
7	a	If $f(x) = e^x$and $g(x) = e^{-x}$in $[a,b]$ then show that c is the average of a and b using Cauchy's mean value theorem.	3	2	5M
	b	Find Taylor's series expansion of the $f(x) = cosx$ about $x = \frac{\pi}{3}$	3	1	
8	a	If U = $\log(x^3 + y^3 + z^3 - 3xyz)$,then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 U = \frac{-9}{(x+y+Z)^2}$	4	5	5M
	b	If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$ and $u = r\sin\theta\cos\emptyset$, $v = r\sin\theta\sin\emptyset, z = rcos\theta$,then find $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)}$.	4	1	5M
		Or			
9	a	Expand $f(x,y) = \tan\left(\frac{y}{x}\right)$ as a Taylor's series at $(1,1)$.Hence compute $f(1.1,0.9)$.	4	3	5M
	b	Find the minimum value of $x^2 + y^2 + z^2$given that $x + y + z = 3a$.	4	1	5M
10	a	Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dx\,dy\,dz$.	5	5	5M
	b	Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant.	5	1	5M
		Or			
11	a	Evaluate$\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\,dy\,dx$.	5	5	5M
	b	Evaluate$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dy\,dx$,by change of order of integration.	5	5	5M

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